MTH 211 Test 2 **Ordinary Differential Equations**
Fall 2025

Name ______________________

All work must be shown *to receive credit for each problem. Please circle your final answers.*

1. Find the general solution for each of the following :

a) $2\dfrac{d^2y}{dt^2} - 7\dfrac{dy}{dt} + 3y = 0$

$2r^2 - 7r + 3 = 0$

$r_1 = \dfrac{1}{2},\ r_2 = 3$

$y(t) = c_1 e^{\frac{1}{2}t} + c_2 e^{3t}$

b) $\dfrac{1}{4}\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} + 2y = 0$

$y'' + 4y' + 8y = 0$

$r^2 + 4r + 8 = 0$

$r_{1,2} = -2 \pm 2i$

$y(t) = c_1 e^{-2t}\cos 2t + c_2 e^{-2t}\sin 2t$

2. ERROR Find the general solution to the initial value problem

$2y'' + 4y = 0\ ;\ \ y(\pi) = 1,\ \ y'(\pi) = 0$ **.**

$\dfrac{1}{2}y'' + 2y = 0\ ;\ \ y(\pi) = 1,\ \ y'(\pi) = 0$

$y'' + 4y = 0$

$r^2 + 4 = 0$

$r_{1,2} = \pm 2i$

$y(t) = c_1\cos 2t + c_2\sin 2t$

$y'(t) = -2c_1\sin 2t + 2c_2\cos 2t$

$y(t) = \cos 2t$

3. Find the **general** solution to the following using the method of undetermined coefficients.

a) $\dfrac{d^2y}{dt^2} - 2\dfrac{dy}{dt} + y = 2\sin t$

$$r^2 - 2r + 1 = 0$$
$$r_{1,2} = 1$$
$$y_n(t) = c_1 e^t + c_2\, t e^t$$
$$y_p(t) = A\sin t + B\cos t$$
$$y_p'(t) = A\cos t - B\sin t$$
$$y_p''(t) = -A\sin t - B\cos t$$
$$-A\sin t - B\cos t - 2(A\cos t - B\sin t) + A\sin t + B\cos t = 2\sin t + 0\cos t$$
$$y_p(t) = \cos t$$
$$\therefore y(t) = c_1 e^t + c_2\, t e^t + \cos t$$

b) $y'' - 4y' + 7y = 14$

$$r^2 - 4r + 7 = 0$$
$$r_{1,2} = 2 \pm i\sqrt{3}$$
$$y_n(t) = c_1 e^{2t}\cos\sqrt{3}\,t + c_2 e^{2t}\sin\sqrt{3}\,t$$
$$y_p(t) = At^2 + Bt + C$$
$$y_p'(t) = 2At + B$$
$$y_p''(t) = 2A$$
$$2A - 4(2At + B) + 7(At^2 + Bt + C) = 0t^2 + 0t + 14$$
$$y_p(t) = 2$$
$$\therefore y(t) = c_1 e^{2t}\cos\sqrt{3}\,t + c_2 e^{2t}\sin\sqrt{3}\,t + 2$$

4. What would be the value of the Wronskian for a null solution of $c_1 t\ln t + c_2 t^2$?

$$\begin{vmatrix} t\ln t & t^2 \\ \ln t + 1 & 2t \end{vmatrix} = t^2\ln t - t^2$$

5. Find the **general** solution to the following using the method of variation of parameters.

$$\frac{1}{6}y'' + \frac{1}{2}y' + \frac{1}{3}y = \frac{1}{1+e^t}.$$

$$y'' + 3y' + 2y = 0$$

$$r^2 + 3r + 2 = 0$$

$$r_1 = -1,\ r_2 = -2$$

$$y(t) = c_1 e^{-t} + c_2 e^{-2t}$$

$$W\left(e^{-t}, e^{-2t}\right) = \begin{vmatrix} e^{-t} & e^{-2t} \\ -e^{-t} & -2e^{-2t} \end{vmatrix} = -e^{-3t}$$

$$y_p(t) = 6\left(-e^{-t}\int \frac{e^{-2t}\left(\frac{1}{1+e^t}\right)}{-e^{-3t}}dt \ + \ e^{-2t}\int \frac{e^{-t}\left(\frac{1}{1+e^t}\right)}{-e^{-3t}}dt \right)$$

$$y_p(t) = -6e^{-t}\left(-\ln\left(1+e^t\right)\right) + 6e^{-2t}\left(e^t - \ln\left(1+e^t\right)\right)$$

6. We know that the null solution $y_n(t) = c_1e^{-t} + c_2te^{-t}$ and the particular solution $y_p(t) = t+1$ are those of a certain non-homogeneous differential equation with constant coefficients Given the initial conditions $y(0) = 4$ and $y'(0) = 8$, find the full solution.

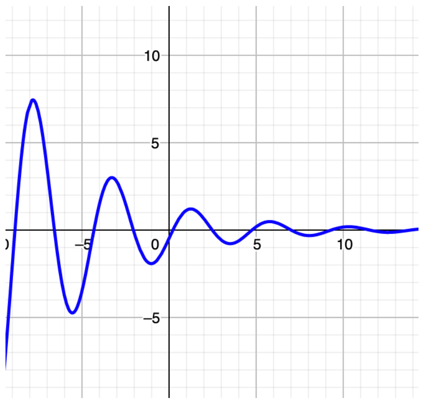
$$y_n(t) = \mathbf{3}e^{-t} + \mathbf{10}\ te^{-t} + t + 1$$

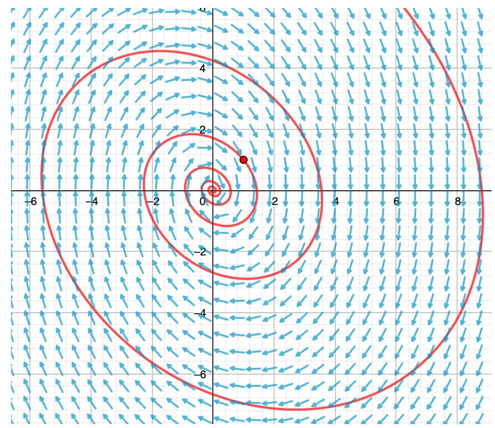
7. To find the angular displacement , $\theta(t)$, at any time t for a linearized pendulum with a mass of 5 kg, a damping constant, $c = 2$, and a length, $L = 5$ m, and external tangential force, $h(t)$, we form the second order linear differential equation

$$\theta''(t) + \frac{c}{m}\theta'(t) + \frac{g}{L}\theta(t) = \frac{1}{mL}h(t) \qquad ($$

Find the angular displacement at any time t where $\theta(0) = 1$ and $\theta'(0) = 1$, and $h(t) = 0$.

(Use $g =$ 9.8 m/s^2)





$$\theta'' + \frac{2}{5}\theta' + \frac{9.8}{5}y = 0 \Rightarrow \theta'' + \frac{2}{5}\theta' + \frac{9.8}{5}y = 0$$

$$25r^2 + 10r + 49 = 0$$

$$r_{1,2} = -\frac{1}{5} \pm \frac{4}{5}\sqrt{3}\, i$$

$$y(t) = c_1 e^{-\frac{1}{5}t}\cos\frac{4}{5}\sqrt{3}\,t + c_2 e^{-\frac{1}{5}t}\sin\frac{4}{5}\sqrt{3}\,t$$

$$y'(t) = -\frac{1}{5}c_1 e^{-\frac{1}{5}t}\cos\frac{4}{5}\sqrt{3}\,t - \frac{4}{5}\sqrt{3}\,c_1 e^{-\frac{1}{5}t}\sin\frac{4}{5}\sqrt{3}\,t -$$

$$-\frac{1}{5}c_2 e^{-\frac{1}{5}t}\sin\frac{4}{5}\sqrt{3}\,t + \frac{4}{5}\sqrt{3}c_2 e^{-\frac{1}{5}t}\cos\frac{4}{5}\sqrt{3}\,t$$

$$c_1 = 1, \;\; c_2 = \frac{1}{2}\sqrt{3}$$

$$y(t) = e^{-\frac{1}{5}t}\cos\frac{4}{5}\sqrt{3}\,t + \frac{1}{2}\sqrt{3}\,e^{-\frac{1}{5}t}\sin\frac{4}{5}\sqrt{3}\,t$$

8. Find the **null** solution, **particular** solution, and **general** solution to $y''' - 2y'' + y' = t$

$$r^3 - 2r^2 + r = 0 \quad = r(r-1)(r-1)$$

$$r_1 = 0,\ r_{2,3} = 1$$

$$y(t) = c_1 + c_2 e^t + c_3 t e^t$$

$$y_p(t) = At^3 + Bt^2 + Ct + D$$

$$y_p'(t) = 3At^2 + 2Bt + C$$

$$y_p''(t) = 6At + 2B$$

$$y_p'''(t) = 6A$$

$$6A - 2(6At + 2B) + 3At^2 + 2Bt + C = 0t^3 + 0t^2 + 1t + 0$$

$$y_p(t) = \frac{1}{2}t^2 + 2t$$

$$\therefore \quad y(t) = c_1 + c_2 e^t + c_3 t e^t + \frac{1}{2}t^2 + 2t$$